Passivity Analysis and Passivation of Interconnected Event-Triggered Feedback Systems Using Passivity Indices ⋆

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Abstract: In this paper, passivity and passivation problems for event-triggered feedback interconnected systems are addressed. We consider passivity in two event-triggered control schemes based on the location of the event-triggered samplers: sampler at plant output and sampler at controller output. For both schemes, we first derive the conditions to characterize the level of passivity for the interconnected system using passivity indices. The event-triggering condition proposed guarantees that these indices can be achieved. Then the passivation problem is considered and passivation conditions are provided. The passivation conditions depend on the passivity indices of the plant and controller and also the event-triggering condition, which reveals the trade off between performance (passivity level) and communication resource utilization.

Keywords: Passivity-based control; Lyapunov methods; Networked systems

1. INTRODUCTION

The notion of dissipativity, and its special case of passivity, are characterizations of system input and output behavior based on a generalized notion of energy. The ideas of passivity first emerged from the phenomenon of dissipation of energy across passive components in the circuit theory field, see e.g. Anderson and Vongpanitlerd [1973]. Passive systems can be viewed as systems that do not generate energy, but only store or release the energy which was provided. Dissipativity was introduced and formalized by Willems [1972], and it is a generalized notion of passivity. Dissipativity and passivity can be applied to the analysis of chemical, mechanical, electromechanical and electrical systems where the definition of energy has both clear physical meaning and concrete mathematical representation. Over the past decades, dissipativity and passivity have received constantly high attention by the systems and control community with plenty of applications in theory and practice, see e.g. Bao and Lee [2007], Khalil [2002], Ebenbauer et al. [2009]. The significant benefit of passivity is that when two passive systems are interconnected in parallel or in feedback, the overall system is still passive. Thus passivity is preserved when large-scales systems are combined from components of passive subsystems. Such compositional property is often used in large-scale network design of nonlinear interconnected systems and related topics, see e.g. Arcak [2007]. The advantage of using this property is that one can always guarantee passivity of the interconnected passive systems and thus stability of the whole system is guaranteed. Recent results in Antsaklis et al. [2012, 2013] also showed its power in compositional design of cyber-physical systems.

Although passivity theory has been applied successfully in control design, this property is vulnerable to discretization, quantization and other factors introduced by digital controllers or communication channels in modern control systems. Results in the literature mainly considered passivity analysis and passivation for a *single* dynamical system under different network effects. Oishi [2010] pointed out that passivity is not preserved under discretization and then quantified how much passivity is lost under standard discretization. For quantization effects, passivity analysis and passivation of LTI systems with quantization were treated as uncertainties described by integral quadratic constraints in Xie et al. [1998]. Recent work by Zhu et al. [2012] derived the conditions under which the passive structure of an output strictly passive (OSP) nonlinear system can be preserved under quantization. On the other hand, it is also important to study passivity and passivation of *interconnected* systems, considering the advantages in the analysis and design of large-scale interconnected systems. As the extension to the well-known compositional property of passivity, Zhu and Antsaklis [2014] considered the passivity and passivation problems for feedback interconnection of two input feed-forward output-feedback (IF-OF) passive systems. Wang et al. [2012] considered passivity analysis for discrete-time periodically controlled nonlinear systems, where the system switches between open and closed loop periodically.

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Motivated by the work in Yu and Antsaklis [2013] and Zhu and Antsaklis [2014], in this paper we consider the passivity and passivation problems for event-triggered feedback interconnected systems. Instead of stability studied in Yu and Antsaklis [2013], we focus on passivity properties of the interconnected system. Based on the location of event-triggered sampler implemented, we have two event-triggered control schemes to consider: event-triggered sampler at the plant output (Fig. 1) and event-triggered sampler at the controller output (Fig. 2). For each control scheme, the condition to characterize the level of passivity for the interconnected system using passivity indices is derived. Event-triggering conditions are proposed to guarantee that these indices can be achieved. For the passivation problem, the condition to render the interconnected system passive is given. The condition depends on the passivity indices of the plant and controller and the event-triggering condition. Moreover, we discuss the trade off between performance (passivity level) and resource utilization by choosing appropriate passive controllers and event-triggering conditions. The results presented in this paper are extensions of the corresponding results in Zhu and Antsaklis [2014], by considering, in addition, the effect of event-triggered samplers.

The paper is organized as follows. In Section 2, we introduce some background on dissipativity/passivity theory and passivity indices. The passivity analysis and passivation problems are stated in Section 3. Section 4 considers the two problems for feedback interconnected systems with event-triggered samplers. Based on the location of the event-triggered samplers, two event-triggered control schemes are considered, namely event-triggered sampler at the plant output and event-triggered sampler at the controller output. The conclusion is provided in Section 5.

2. PRELIMINARIES AND BACKGROUND

We first introduce some basic concepts of passive and dissipative system theory. Consider the following nonlinear system G, which is driven by an input $u(t)$ and has an output $y(t)$

$$G : \begin{cases} \dot{x}(t) = f\left(x(t), u(t)\right) \\ y\left(t\right) = h\left(x(t), u(t)\right) \end{cases} \tag{1}$$

where $x\left(t\right) \in \mathcal{X} \subset \mathbb{R}^n$, $u\left(t\right) \in \mathcal{U} \subset \mathbb{R}^m$ and $y\left(t\right) \in \mathcal{Y} \subset \mathbb{R}^p$ are the state, input and output of the system respectively and $\mathcal{X}$, $\mathcal{U}$ and $\mathcal{Y}$ are the state, input and output spaces, respectively.

The definition of a dissipative system is based on a storage function (energy stored in the system) and a supply function (externally supplied energy). The basic idea behind dissipativity is that the increase of the stored energy is bounded by the supplied energy.

Definition 1. (Ebenbauer et al. [2009]). System G is said to be dissipative with respect to the supply rate $\omega(x, u, y)$, if there exists a positive semi-definite storage function $V(x)$ such that the (integral) dissipation inequality

$$V(x(t_1)) - V(x(t_0)) \leq \int_{t_0}^{t_1} \omega(x(t), u(t), y(t))dt \tag{2}$$

is satisfied for all t_0, t_1 with $t_0 \leq t_1$ and all solutions $x = x(t), u = u(t), y = y(t)$, $t \in [t_0, t_1]$. If the storage function is differentiable, then the integral dissipation inequality (2) can be rewritten as

$$\dot{V}(x(t)) \leq \omega(x(t), u(t), y(t)), \forall t \tag{3}$$

As a special case of dissipativity, *QSR-dissipativity* was proposed in Hill and Moylan [1976]. In this case the supply rate is chosen to be

$$\omega(u, y) = y^T Qy + 2y^T Su + u^T Ru \tag{4}$$

where Q, S and R are matrices with appropriate dimensions. The relation between QSR-dissipativity and $\mathcal{L}_2$ stability has been shown in Hill and Moylan [1976].

Theorem 2. (Hill and Moylan [1976]). If System G is QSR-dissipative with $Q < 0$, then it is $\mathcal{L}_2$ stable.

Definition 3. (Bao and Lee [2007]). System G with $m = p$ is passive if there exists a positive semi-definite storage function $V(x)$ such that the following inequality holds for all $t_1, t_2 \in [0, \infty)$ such that

$$V(x(t_2)) - V(x(t_1)) \leq \int_{t_1}^{t_2} u(t)^T y(t)dt \tag{5}$$

If the storage function is smooth, then the integral dissipation inequality (5) can be rewritten as $\dot{V}\left(x\left(t\right)\right) \leq u^T y$.

In order to measure the excess and shortage of passivity, passivity indices (or passivity levels) were introduced, see e.g. Khalil [2002], Bao and Lee [2007]. The indices can be used to render the system passive with feedback and feed-forward compensation; they can also used to describe the performance of passive systems.

Definition 4. (Khalil [2002]). A system is input feed-forward output feedback passive (IF-OFP) if it is dissipative with respect to the supply rate

$$\omega(u, y) = u^T y - \nu u^T u - \rho y^T y,\ \forall t \geq 0, \tag{6}$$

for some $\rho, \nu \in \mathbb{R}$.

Based on Definition 4, we can denoted an IF-OFP system by IF-OFP(ν, ρ). Definition 4 is often used in passivity analysis, passivation and passivity-based control, see e.g. Xia et al. [2013], Yu and Antsaklis [2013], Zhu et al. [2012], Oishi [2010]. It can be seen that when $\rho = \nu = 0$ an IF-OFP system is simply a passive system. one can further have the definitions of input feed-forward (strictly) passive, output feedback (strictly) passive and very strictly passive.

(1) When $\rho = 0$ and $\nu \neq 0$, the system is said to be input feed-forward passive (IFP), denoted as IFP(ν). when in addition $\nu > 0$, the system is input feed-forward strictly passive (ISP).
(2) When $\rho \neq 0$ and $\nu = 0$, the system is said to be output feedback passive (OFP), denoted as OFP(ρ). When in addition $\rho > 0$, the system is output feedback strictly passive (OSP).
(3) When $\rho > 0$ and $\nu > 0$, the system is said to be very strictly passive (VSP).

Note that positive ρ or ν means that the system has an excess of passivity, such as ISP, OSP and VSP. If either ρ or ν is negative, the system has a shortage of passivity and thus is non-passive. When one of indices is zero and the other is non-zero (i.e. IFO and OFP), ρ or ν is called "*passivity index*", defined as the largest value such that (6) holds for $\forall u$ and $\forall t \geq 0$ (See Bao and Lee [2007]). When

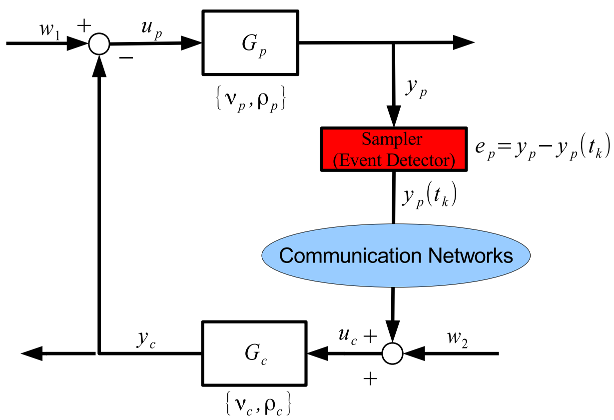

Fig. 1. Feedback connection of two IF-OFP systems with event-triggered sampler at plant output

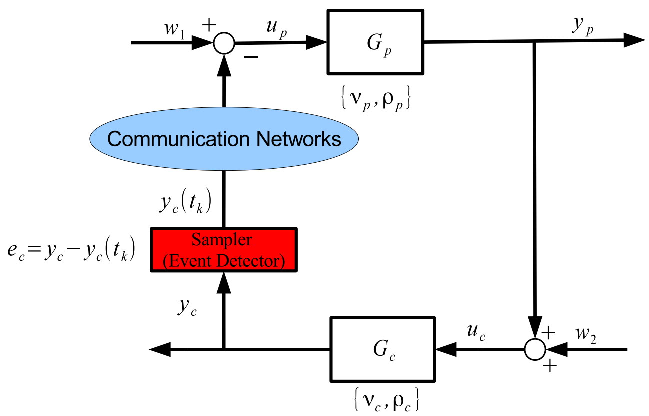

Fig. 2. Feedback connection of two IF-OFP systems with event-triggered sampler at controller output

both of indices are non-zero, the values of ρ and ν may not be unique and are sometimes referred as "*passivity levels*" (see Xia et al. [2013]). In this paper, *we do not distinguish between these two notions as long as there exist ρ and ν such that* (6) *holds.*

The valid domain of ρ and ν has been proposed in Matiakis et al. [2006], Yu et al. [2013].

Lemma 5. (Yu et al. [2013]). The domain of ρ and ν in IF-OFP system is $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \left\{\rho, \nu \in \mathbb{R} | \rho\nu < \frac{1}{4}\right\}$ and $\Omega_2 = \left\{\rho, \nu \in \mathbb{R} | \rho\nu = \frac{1}{4}; \rho > 0\right\}$.

In this paper, we adopt Definition 4 and assume that ρ and ν are in the domain unless otherwise noted.

3. PROBLEM FORMULATION

We first consider feedback connection of two systems with an event-triggered sampler at the plant output, given in Fig. 1. We assume G_p is IF-OFP(ν_p, ρ_p) and G_c is IF-OFP(ν_c, ρ_c) with known passivity indices. Instead of assuming continuous communication in the feedback loop as in Zhu and Antsaklis [2014], an event-triggered feedback scheme is introduced. Event-triggered control has been introduced for the possibility of reducing resources usage (i.e., sampling rate, CPU time, network access frequency), see e.g. Aström [2008], Donkers and Heemels [2012], Heemels et al. [2008], Lemmon [2010], Otanez et al. [2002], Mazo and Tabuada [2008], Tabuada [2007], Yu and Antsaklis [2013]. The triggering mechanisms are referring to the situation in which the control signals are kept constant until the violation of a condition on certain signals triggers the re-computation of the control signals. As in Fig. 1, the new output information of G_p is sent to the controller G_c only when the output novelty error $e_p = y_p - y_p(t_k)$ in the event-triggered sampler satisfies a triggering condition. $y_p(t_k)$ denotes the last output information sent to the controller G_c at the event time t_k. Note that Yu and Antsaklis [2013] considered the same control scheme but focused on deriving the triggering condition to guarantee *stability* of the closed-loop system. In the present paper, we focus on characterizing *dissipativity/passivity* properties of the closed-loop system, which can be viewed as extensions of the results in Yu and Antsaklis [2013] and Zhu and Antsaklis [2014]. The main problems investigated in the present paper are summarized as follows.

(1) Given the passivity indices of G_c and G_p, how can we determine the passivity indices for the closed-loop systems and accordingly, what is the event-triggering condition to guarantee that these indices can be achieved?
(2) For a non-passive plant G_p and a passive controller G_c, what condition on the passivity indices of both systems should be satisfied to render the closed-loop system passive and accordingly, what is the event-triggering condition to guarantee that the condition can be satisfied?

In addition to feedback connection with an event-triggered sampler of plant output, another similar scheme can be considered as in Fig. 2, where the event-triggered sampler is implemented in the output path of the controller G_c. The new output information of G_c is sent to the plant G_p only when the output novelty error $e_c = y_c - y_c(t_k)$ in the event-triggered sampler satisfies a triggering condition. $y_c(t_k)$ denotes the last output information sent to the controller G_c at the event time t_k. Analogously, same questions listed above also need to be considered and answered.

4. MAIN RESULTS

In this section, we consider the passivity analysis and passivation problems (two problems proposed in Section 3) for event-triggered feedback interconnected systems using passivity indices. Based on the location of event-triggered sampler implemented, we have two event-triggered control schemes to consider: sampler at the plant output and sampler at the controller output. For both schemes, we first derive the conditions to characterize the level of passivity for the closed-loop system using passivity indices. Then the passivation problem is considered and the passivation conditions are provided. Some results are given without proofs. One can refer to Zhu et al. [2013] for the complete proofs.

4.1 Passivity Analysis and Passivation for Event-Trigerred Sampler at Plant Output

We first consider the passivity analysis problem for the feedback system with an event-triggered sampler of the plant output (Fig. 1). Lemma 6 relates the interconnected system to QSR-dissipative systems.

Lemma 6. Consider the feedback interconnection of two IF-OF systems with the passivity indices ν_p, ρ_p and ν_c, ρ_c respectively (Fig. 1). If the event time t_k is explicitly determined by the following triggering condition

$$\|e_p(t)\|_2 = \frac{\beta_p}{\sqrt{\nu_c^2 + m_p\beta_p} + |\nu_c|}\,\|y_p(t)\|_2 \tag{7}$$

where $m_p = \frac{1}{4\alpha_p} + |\nu_c| - \nu_c$, $\alpha_p > 0$ and $\beta_p > 0$, then the interconnected system is QSR-dissipative (with respect to the input $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$ and output $y(t) = \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix}$), satisfying the inequality

$$\dot{V}(t) \leq y(t)^T Q y(t) + 2w(t)^T S y(t) + w(t)^T R w(t) \tag{8}$$

where

$$Q = \begin{bmatrix} -(\rho_p + \nu_c - \beta_p) I & 0I \\ 0I & -(\nu_p + \rho_c - \alpha_p) I \end{bmatrix},$$

$$S = \begin{bmatrix} \frac{1}{2}I & \nu_p I \\ -\nu_c I & \frac{1}{2}I \end{bmatrix},$$

and

$$R = \begin{bmatrix} -\nu_p I & 0I \\ 0I & -(\nu_c - |\nu_c|) I \end{bmatrix}.$$

Proof. See the proof of Lemma 8 in Zhu et al. [2013].

Remark 7. Although Lemma 6 does not explicitly characterize passivity indices for the closed-loop system, it determines an event-triggering condition (7) which guarantees that the closed-loop system is QSR-dissipative. After preserving QSR-dissipativity of the closed-loop system, same proof techniques used in Zhu and Antsaklis [2014] can be applied to further explore passivity properties of the system.

Remark 8. As pointed out in Theorem 2, the closed-loop system (Fig. (1)) is $\mathcal{L}_2$ stable if $Q < 0$. It can be seen that a sufficient condition for $Q < 0$ is $\nu_p + \rho_c > \alpha_p$ and $\nu_c + \rho_p > \beta_p$, which is similar to the condition derived in Yu and Antsaklis [2013]. Also note that the triggering condition here is different from the condition in Yu and Antsaklis [2013].

Remark 9. It can be seen from (7) that larger α_p and β_p result in a larger triggering threshold. A large triggering threshold implies lower sampling rate and thus lower resources usage. Later we will show how these parameters affect passivity of the system.

Next, Theorem 10 shows how to determine the passivity indices for the feedback system with event-triggered sampler of plant output.

Theorem 10. Consider the feedback interconnected system in Fig. 1. Suppose the passivity indices ν_p, ρ_p, ν_c and ρ_c are known and the triggering condition is determined by (7). If we choose ϵ and δ such that

$$\begin{cases} \epsilon < \min\{\nu_p, \nu_c - |\nu_c|\} \\ \delta \leq \min\left\{\rho_c - \alpha_p - \dfrac{\epsilon\nu_p}{\nu_p - \epsilon}, \rho_p - \beta_p - \dfrac{(|\nu_c| + \epsilon)\nu_c}{\nu_c - |\nu_c| - \epsilon}\right\} \end{cases}, \tag{9}$$

then the interconnected system has passivity indices ϵ and δ satisfying

$$\dot{V} \leq w^T(t)y(t) - \epsilon w^T(t)w(t) - \delta y^T(t)y(t) \tag{10}$$

where $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$ and $y = \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix}$.

Proof. See the proof of Theorem 12 in Zhu et al. [2013].

Remark 11. (9) can be used to obtain an estimate of the passivity indices for the closed-loop system, with respect to the input $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ and output $y = \begin{bmatrix} y_p \\ y_c \end{bmatrix}$. The condition is similar to its counterpart in Zhu and Antsaklis [2014]. Additionally, (9) quantifies the impact of triggering condition on the passivity indices of the closed-loop system using the parameters α_p and β_p.

Now we introduce the passivation problem for the feedback system with event-triggered sampler of plant output. For this problem, the goal is to passivate a non-passive plant G_p using a passive controller G_c. Here passivity of the interconnected system is defined on the input w_1 and output y_p. We also assume that w_2 is zero. One may observe from Theorem 9 that passivity with respect to the full input and output (i.e. input w and output y) may not be guaranteed to be reinforced under feedback interconnection and event-triggering scheme. However, since we have selected different inputs and outputs, the corresponding passivity may change accordingly. Theorem 12 shows that it is possible to guarantee passivity for the desired input and output although passivity for full input and output may not hold.

Theorem 12. Assume $w_2 = 0$ and let the triggering condition be determined by (7). The interconnected system (Fig. (1)) is passive with respect to the input w_1 and output y_p if the passivity indices satisfy the conditions

$$\nu_p \geq 0 \tag{11}$$

$$\rho_c \geq \alpha_p \tag{12}$$

$$\rho_p + \nu_c \geq \beta_p. \tag{13}$$

Proof. See the proof of Theorem 14 in Zhu et al. [2013].

Remark 13. When the plant G_p is non-passive (i.e. $\rho_p < 0$), the closed-loop system can be rendered passive by choosing a passive controller G_c with $\rho_c \geq \alpha_p$ and $\nu_c \geq -\rho_p + \beta_p$. Compared with the passivation conditions in Zhu and Antsaklis [2014], the conditions (11)-(13) imply that one needs a passive controller with higher passivity indices to passivate a non-passive plant for a triggering condition with fixed α_p and β_p. On the other hand, the conditions also give the upper bounds for α_p and β_p to guarantee closed-loop passivity for a given plant and controller with known passivity indices. The results provide certain flexibility for designers by trade off between passivity level of the controller and resource utilization.

Moreover, we can also obtain an estimate of passivity indices for the passivated system, as shown in Corollary 14.

Corollary 14. Assume the triggering condition is determined by (7). Suppose that the conditions (11)-(13) are satisfied and $\nu_p + \rho_c > \alpha_p$. If we choose ϵ and δ such that

$$\begin{cases} 0 \leq \epsilon \leq \dfrac{\nu_p(\rho_c - \alpha_p)}{\nu_p + \rho_c - \alpha_p} \\ 0 \leq \delta \leq \nu_c + \rho_p - \beta_p \end{cases}, \tag{14}$$

then the interconnected system (Fig. 1) has passivity indices ϵ and δ satisfying

$$\dot{V}(t) \le w_1^T(t)y_p(t) - \epsilon w_1^T(t)w_1(t) - \delta y_p^T(t)y_p(t) \quad (15)$$

Proof. See the proof of Corollary 16 in Zhu et al. [2013].

Remark 15. Because of the conditions (11)-(13) and $\nu_p + \rho_c > \alpha_p$, the passivity indices ϵ and δ are upper bounded by positive numbers. (14) provides a way to obtain the desired passivity indices of the closed-loop system by choosing a passive G_c with proper indices and a triggering condition with proper α_p and β_p. As we point out in Remark 13, the trade off between performance (passivity level) and resource utilization can be considered. For instance, if an OFP index given by $\delta = \nu_c + \rho_p - \beta_p$ is desired, one can either choose a passive controller with high ν_c and a triggering condition with low β_p to conserve more communication resources, or a triggering condition with high β_p and a passive controller with low ν_c to impose less restrictions on the controller design.

4.2 Passivity Analysis and Passivation for Event-Trigerred Sampler at Controller Output

For the feedback system with event-triggered sampler at the controller output (Fig. 2), we can follow the same rationale as for the feedback system with event-triggered sampler at the plant output. We first consider the passivity analysis problem and then move to the passivation problem.

Lemma 16. Consider two IF-OF systems with the passivity indices ν_p, ρ_p and ν_c, ρ_c respectively. If the event time t_k is explicitly determined by the following triggering condition

$$\|e_c(t)\|_2 = \frac{\beta_c}{\sqrt{\nu_p^2 + m_c\beta_c} + |\nu_p|} \|y_c(t)\|_2 \quad (16)$$

where $m_c = \frac{1}{4\alpha_c} + |\nu_p| - \nu_p$, $\alpha_c > 0$ and $\beta_c > 0$, then the interconnected system with the event-triggered sampler (Fig. 2) is QSR-dissipative (with respect to the input $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$ and output $y(t) = \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix}$), which satisfies the inequality

$$\dot{V}(t) \le y(t)^T Q y(t) + 2w(t)^T S y(t) + w(t)^T R w(t) \quad (17)$$

where

$$Q = \begin{bmatrix} -(\rho_p + \nu_c - \alpha_c) I & 0I \\ 0I & -(\nu_p + \rho_c - \beta_c) I \end{bmatrix},$$

$$S = \begin{bmatrix} \frac{1}{2}I & \nu_p I \\ -\nu_c I & \frac{1}{2}I \end{bmatrix},$$

and

$$R = \begin{bmatrix} -(\nu_p - |\nu_p|) I & 0I \\ 0I & -\nu_c I \end{bmatrix}.$$

Proof. See the proof on Lemma 18 in Zhu et al. [2013].

Theorem 17. Suppose that the passivity indices ν_p, ρ_p, ν_c and ρ_c are known and the triggering condition is determined by (16). If we choose ϵ and δ such that

$$\begin{cases} \epsilon < \min\{\nu_p - |\nu_p|, \nu_c\} \\ \delta \le \min\left\{\rho_p - \alpha_c - \dfrac{\epsilon\nu_c}{\nu_c - \epsilon}, \rho_c - \beta_c - \dfrac{(|\nu_p| + \epsilon)\nu_p}{\nu_p - |\nu_p| - \epsilon}\right\} \end{cases}, \quad (18)$$

the interconnected system with the event-triggered sampler (Fig. 2) has the passivity indices ϵ and δ satisfying

$$\dot{V} \le w^T(t)y(t) - \epsilon w^T(t)w(t) - \delta y^T(t)y(t) \quad (19)$$

where $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$ and $y = \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix}$.

Proof. See the proof of Theorem 19 in Zhu et al. [2013].

Remark 18. The results in Lemma 16 and Theorem 17 are similar to their counterparts for the feedback system with event-triggered sampler of plant output. However, note that the triggering condition (16) now depends on α_c, β_c and ν_p. Moreover, the matrices Q, S and R in (17) are different from those in (8).

For the passivation problem, Theorem (19) gives the conditions of rendering the interconnected system passive.

Theorem 19. Assume $w_2 = 0$ and the triggering condition is determined by (16). The interconnected system with the event-triggered sampler (Fig. 2) is passive with respect to the input w_1 and output y_p if the passivity indices satisfy the conditions

$$\nu_p = 0 \quad (20)$$

$$\rho_c \ge \beta_c \quad (21)$$

$$\rho_p + \nu_c \ge \alpha_c. \quad (22)$$

Proof. See the proof of Theorem 21 in Zhu et al. [2013].

Remark 20. The condition (20) requires the plant G_p to be a OFP system. Because of (20), the triggering condition (16) can be further simplified as $\|e_c(t)\|_2 = 2\sqrt{\alpha_c\beta_c}\,\|y_c(t)\|_2$, which shows that the triggering condition is independent of the passivity indices of the plant G_p and controller G_c. Therefore, one can first design a desired triggering condition by choosing α_c and β_c, and then design a passive controller satisfying the conditions (20)-(22), or vice versa.

Corollary 21. Suppose that the conditions (20)-(22) are satisfied. If we choose ϵ and δ such that

$$\begin{cases} \epsilon = 0 \\ 0 \le \delta \le \rho_p + \nu_c - \alpha_c \end{cases}, \quad (23)$$

the interconnected system with event-triggering (Fig. 2) has the passivity indices ϵ and δ satisfying

$$\dot{V}(t) \le w_1^T(t)y_p(t) - \epsilon w_1^T(t)w_1(t) - \delta y_p^T(t)y_p(t) \quad (24)$$

Proof. See the proof of Corollary 23 in Zhu et al. [2013]

Remark 22. The condition (23) implies that the closed-loop system is actually an OSP system with an OFP index $\delta \le \rho_p + \nu_c - \alpha_c$. The ideas of passivity indices design and passivity-resource trade off discussed in Remark (15) apply likewise.

5. CONCLUSION

In this paper, we considered the problems of passivity analysis and passivation using passivity indices for interconnected event-triggered feedback systems. The present work extended our previous work in Zhu and Antsaklis [2014] for feedback interconnected systems assuming continuous communication in the feedback loop. We considered two event-triggered control schemes: an event-triggered sampler at the plant output and an event-triggered sampler at the controller output. Using the passivity indices of the plant and controller, the conditions to determine the passivity indices of the interconnected system were given, under a proposed event-triggering condition. We also showed the passivation conditions in terms of the passivity indices of the plant and controller and the triggering condition. The trade off between passivity and communication resources utilization was also discussed. Simulation results can be found in Zhu et al. [2013].

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